COMPRESSION SHOCK IN A TWO-PHASE MEDIUM

L. M. Zolotarev and I. D. Samsonov

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 4, pp. 95–99, 1969

We examine a normal compression shock in an aerosol, of which one phase is a viscous gas and the other consists of solid particles. We study the continuous solution with account for force interaction between the phases. Several versions are calculated numerically for the isothermal and isentropic approximations.

The Rakhmatulin equations [1] are used to describe the process.

An examination of strong disturbances in an n-component medium using the Rakhmatulin equations was made in [2], where the discontinuous solution was investigated.

We consider the continuous solution for a medium with known interaction law between the components; the equations of motion are transformed correspondingly. For an analogous medium the piston and rarefaction-wave problems were solved in [3]. Therein the solution was obtained by the method of characteristics for low-intensity disturbances for Stokes'-law interaction between the particles and the gas.

In contrast with [3], in the present study the force interaction is taken in more general form with use of the resistance coefficient c_x, which makes it possible to examine the process for both small and large relative velocities. The study is made in a coordinate system fixed with the compression shock front.

The Rakhmatulin equations in the one-dimensional case have the form [1]

$$\frac{\partial \rho_n}{\partial t} + \operatorname{div}(\rho_n u_n) = \sum_{j=1}^{n} \lambda_{jn}\rho_j - \rho_n \sum_{j=1}^{n} \lambda_{nj}$$

$$\frac{du_n}{dt} = -\frac{1}{\rho_{ni}}\frac{\partial p}{\partial x} + \sum_{j=1}^{n} \frac{K_{jn}}{\rho_n}(u_j - u_n) + X_n \tag{1}$$

$$p = p(\rho_{ni}, C_n), \qquad \frac{\rho_1}{\rho_{1i}} + \cdots + \frac{\rho_n}{\rho_{ni}} = 1$$

When using equations of this type to study different media, in each particular case we must concretize the transformation laws and the force interaction laws.

Assuming that the principles of continuum mechanics are valid for the aerosol and that the particles are spherical, and considering the magnitude of the interaction force from the gas to be a function of the relative velocity squared, then for the motion without mutual transformations of the components and with account for the gas viscosity we have

$$\frac{\partial \rho_1}{\partial t} + \operatorname{div}(\rho_1 u_1) = 0, \qquad \frac{\partial \rho_2}{\partial t} + \operatorname{div}(\rho_2 u_2) = 0$$

$$\frac{du_1}{dt} = -\frac{1}{\rho_{1i}}\frac{\partial p}{\partial x} + \frac{3}{8}\frac{c_x}{r}\frac{\rho_2\rho_{1i}}{\rho_1\rho_{2i}}(u_1 - u_2)^2 + \frac{4}{3\rho_{1i}}\frac{\partial}{\partial x}\left[\mu\frac{\partial u_1}{\partial x}\right] \tag{2}$$

$$\frac{du_2}{dt} = -\frac{3}{8}\frac{c_x}{r}\frac{\rho_{1i}}{\rho_{2i}}(u_1 - u_2)^2$$

$$p = p(\rho_{1i}, C), \qquad \frac{\rho_1}{\rho_{1i}} + \frac{\rho_2}{\rho_{2i}} = 1$$

The derivation of the equations is not presented, since this would be a repetition of the basic aspects of [1].

The resistance coefficient c_x is a function of the Reynolds number, and the subscripts 1 and 2 denote the gas and solid phase, respectively. Interaction of the particles with one another is assumed to be negligible.

Equations (2) are used to study the compression shock of constant intensity (with constant parameters ahead of the shock); the equations are rewritten for the stationary case in a coordinate system fixed with the shock front:

$$\frac{d}{dx}(\rho_1 u_1)=0, \quad \frac{d}{dx}(\rho_2 u_2)=0, \quad \frac{\rho_1}{\rho_{1i}}+\frac{\rho_2}{\rho_{2i}}=1$$
$$u_1\frac{du_1}{dx}=-\frac{1}{\rho_{1i}}\frac{dp}{dx}+\frac{3}{8}\frac{c_x}{r}\frac{\rho_2\rho_{1i}}{\rho_1\rho_{2i}}(u_1-u_2)^2+\frac{4}{3\rho_{1i}}\frac{d}{dx}\left[\mu\frac{du_1}{dx}\right] \tag{3}$$
$$u_2\frac{du_2}{dx}=-\frac{3}{8}\frac{c_x}{r}\frac{\rho_{1i}}{\rho_{2i}}(u_1-u_2)^2, \quad p=p(\rho_{1i}, C)$$

In the case of motion of a gas without particles, even with a shock wave amplitude which varies with time, the behavior and structure of the shock front can be described for any instant with the aid of steady-state theory.

This is possible because over time intervals small in comparison with the over-all time scale of the gasdynamic process but longer than the time Δt for the front to travel a distance on the order of its width Δx the entire distribution pattern of the quantities in the wave front propagates through the gas in "frozen" form as a whole [4, 5].

For the medium considered in the present study with its own sort of macroscopic-scale relaxation, the use of steady-state theory to study unsteady processes is limited. In this case the acceptable rate of change of the parameters ahead of the front can be estimated directly from the solution of (3) after determining the magnitude of the relaxation zone.

Thus, we examine the case of small volumetric content of the suspended particles, i.e., $\rho_1/\rho_{1i} \sim 1$. With account for this, after partial integration (3) takes the form

$$\rho_1 u_1=\rho_1^\circ u_1^\circ, \quad \rho_2 u_2=\rho_2^\circ u_2^\circ, \quad p=p(\rho_{1i}, C)$$
$$\rho_1^\circ u_1^\circ u_1+\rho_2^\circ u_2^\circ u_2+p-\frac{4}{3}\mu\frac{du_1}{dx}=\rho_1^\circ u_1^{\circ 2}+\rho_2^\circ u_2^{\circ 2}+p^\circ \tag{4}$$
$$u_2\frac{du_2}{dx}=-\frac{3}{8}\frac{c_x}{r}\frac{\rho_{1i}}{\rho_{2i}}(u_1-u_2)^2$$

For further transformations of (4) it is advisable to introduce the dimensionless quantities

$$U_1=\frac{u_1}{u_1^\circ}, \quad U_2=\frac{u_2}{u_2^\circ}, \quad X=\frac{x}{r}, \quad R^\circ=\frac{2u_1^\circ r}{\nu^\circ}$$
$$E_1^\circ=\frac{p^\circ}{\rho_1^\circ u_1^{\circ 2}}, \quad E_2^\circ=\frac{p^\circ}{\rho_2^\circ u_2^{\circ 2}}, \quad E_3^\circ=\frac{p^\circ}{\rho_{2i} u_2^{\circ 2}}$$

In the present study the primary question is force interaction of the phases, and therefore in evaluating the order of magnitudes and examining the qualitative aspect in the energetics of the process it is sufficient to limit ourselves to certain approximations.

As these approximations we use the isothermal (μ = const, $p = p^\circ\rho_1/\rho_1^\circ$) and isentropic (the Poisson adiabat for the gaseous phase with thermally isolated particles) assumptions.

In the isothermal case with account for the adopted notations, we write (4) after simple transformations as

$$\frac{dU_1}{dX}=\frac{3}{8}R^\circ\left(U_1+\frac{E_1^\circ}{E_2^\circ}U_2+\frac{E_1^\circ}{U_1}-1-\frac{E_1^\circ}{E_2^\circ}-E_1^\circ\right)$$
$$\frac{dU_2}{dX}=\frac{3}{8}\frac{E_3^\circ}{E_1^\circ}c_x\left(2-\frac{U_1}{U_2}-\frac{U_2}{U_1}\right) \tag{5}$$

The isentropic case differs formally from the isothermal case in that in the first equation of (5) the third term in the parentheses takes the form E_1°/U_1^k and R° = var.

For simplicity we assume that the phase velocities are the same at the initial time. Then to describe the compression shock we must combine with (5) the boundary conditions, which express the absence of gradients ahead of and behind the shock and the approach of the gasdynamic quantities to the initial (as $X \to -\infty$) and final (as $X \to +\infty$) values. Thus, setting in (5) the derivatives of the velocities equal to zero ($dU_1/dX = dU_2/dX = 0$), it is easy to find the velocity boundary conditions

$$X = -\infty, \quad U_1 = U_2 = 1$$
$$X = +\infty, \quad U_1 = U_2 = \frac{E_1^\circ E_2^\circ}{E_1^\circ + E_2^\circ} \tag{6}$$

These values correspond to the usual relations on a discontinuity. If the particle quantity is small ($E_2^\circ \to \infty$), after resolving the ambiguity in (6) we obtain $U = E_1^\circ$, which corresponds to the case of pure gas. We see from (6) that the shock intensity increases, other conditions being the same, with increase of the particle mass, specifically,

$$U = \frac{E_1^\circ}{1 + \rho_1^\circ / \rho_2^\circ} \tag{7}$$

System (5) was solved numerically for the case of a water-air medium (fog) under the assumption that breakup of the droplets and phase transformations do not occur, i.e., the water particles are equivalent to solid bodies. The flow parameters ahead of the shock were

$$p^\circ = 9.81 \cdot 10^4 \, \mathrm{N/m^2}, \quad T = 303^\circ \, \mathrm{K}, \quad \nu^0 = 16 \cdot 10^{-6} \, \mathrm{m^2/sec}, \quad M = 1.1$$
$$\rho_1^\circ = \rho_2^\circ, \quad E_1^\circ = E_2^\circ \approx 0.598, \quad E_3^\circ \approx 0.662 \cdot 10^{-3}$$

We see that the mass relationships for the gas and the particles were assumed to be the same. Evaluation of the ratios of the fictitious and actual gas densities yields

$$\frac{\rho_1}{\rho_{1i}} = \frac{1}{1 + \rho_{1i} / \rho_{2i}} \approx 0.999$$

i.e., the densities are interchangeable to within 0.001.

In solving (5) the particle sizes were taken to be

$$r = 10^{-6}, 10^{-7}, 10^{-8}, 10^{-9} \quad ([r] - \mathrm{m})$$

For the sake of generality we included the case of a shock in the pure gas with the same initial values.

The relation for c_x was taken from [6]:

$$c_x = f(R) \tag{8}$$

The Reynolds number is based on the relative velocity:

$$R = \frac{2r \, |u_1 - u_2|}{\nu} \tag{9}$$

If for low Reynolds numbers we replace c_x by its approximate expression

$$c_x = 24 / R \tag{10}$$

then (5) will describe the case of Stokes interaction between the medium and the particles:

$$\frac{dU_1}{dX} = \frac{3}{8} R^\circ \left(U_1 + \frac{E_1^\circ}{E_2^\circ} U_2 + \frac{E_1^\circ}{U_1} - 1 - \frac{E_1^\circ}{E_2^\circ} - E_1^\circ \right)$$
$$\frac{dU_2}{dX} = \frac{9}{R} \frac{E_3^\circ}{E_1^\circ} \left(1 - \frac{U_1}{U_2} \right) \tag{11}$$

We note that this solution is basically qualitative since the application of the usual laws to the process of flow past very small particles is not rigorously justified.

For the case of a compression shock in pure air it is easy to obtain an analytic solution which can be used for comparison with the numerical results:

$$X = \frac{8}{3R^\circ (1 - E_1^\circ)} \ln \frac{1 - U}{(U - E_1^\circ)^{E_1^\circ}} \tag{12}$$

The calculated data for the two-phase medium are shown graphically in Figs. 1 and 2.

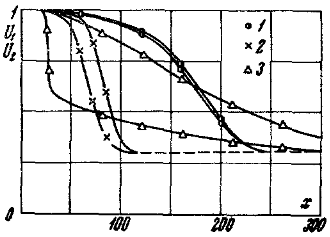

Fig. 1

Figure 1 gives an idea of the qualitative aspect of the process, while Fig. 2 shows the variation of the intensity of the process in the gaseous phase as a function of the particle size (points 1, 2, 3, and 4 indicate, respectively, the computational results for particles with $r = 10^{-9}$ m, 10^{-8} m, 10^{-7} m, 10^{-6} m; 5 (·) corresponds to the case of a compression shock in a pure gas).

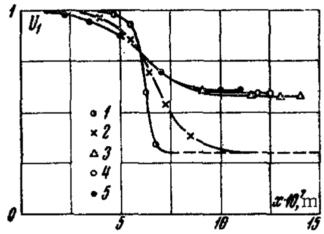

Fig. 2

We see that for comparatively large particles the process breaks down into two segments which are of different orders of magnitude.

Within the limits of the first segment the particle velocity remains practically constant, while in the gas there is a marked velocity change, corresponding in intensity and gradient to the case of a shock in a pure gas.

In the second segment the gas and particle parameters reach their final values in a significantly retarded process. However, we note that on the whole the disturbance region remains small for the case of small particles. With reduction of the particle radius the nature of the shock changes so that the process for both phases becomes of the same order with regard to intensity with a general increase of the velocity gradient. It is obvious that the isothermal model of the compression shock in a viscous gas is thermodynamically quite approximate and therefore must be considered a first approximation, as, for example, in [7].

In our study we examined the isentropic shock as the second approximation. This compression shock model is qualitatively better since in the series expansion the Poisson adiabat coincides with the Hugoniot shock adiabat to within the third-order term [4, 5]. However, since the intermediate states in the case of strong compression shocks are not described by the equation of the adiabats mentioned above, to obtain more accurate results in these cases we must use the energy-balance equation.

The results of the calculation of the isentropic shock with the previous initial conditions are shown in Fig. 3, where points 1 correspond to the case $r = 10^{-8}$ m, points 2 are for the case $r = 10^{-7}$ m, and points 3 are for the case of a shock in the pure gas.

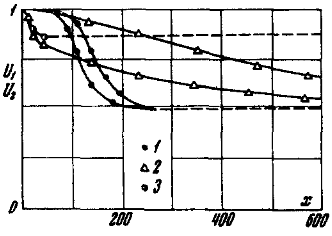

Fig. 3

The viscosity-temperature dependence was taken as

$$\mu = \mu^\circ \left(\frac{T}{T^\circ}\right)^{0.7} \tag{13}$$

We see from Fig. 3 that with regard to the nature of the variation of the phase velocities this case does not represent any qualitatively new phenomenon and does not differ markedly from the isothermal case with respect to the order of the gasdynamic quantities.

In general, if in some process the gas parameters vary in accordance with the Poisson adiabat with exponent k, then the parameters of the two-phase medium vary polytropically with some exponent n, defined in p, v variables as

$$n = -\frac{v}{p}\frac{\partial p}{\partial v} \tag{14}$$

For the medium in question without phase transformation and heat transfer between the phases, the polytropic exponent can be obtained easily in the form

$$n = k\left[1 + \frac{1-\varkappa}{\varkappa}\frac{v_{2i}}{v_{1i}}\left(\frac{p}{p^\circ}\right)^{1/k}\right] \tag{15}$$

With account for heat transfer the expression for the polytropic exponent changes:

$$n = \frac{1}{1-\varkappa R/c}\left[1 + \frac{1-\varkappa}{\varkappa}\frac{v_{2i}}{v_{1i}}\left(\frac{p}{p^\circ}\right)^{1-\varkappa R/c}\right] \tag{16}$$

Thus, if the medium in question is taken to be some hypothetical gas with averaged parameters and if we write the usual relations on the shock, we obtain easily the expression for the shock adiabat in this case:

$$\frac{p}{p^\circ} = \frac{[1 + k(2k/n - 1)]\,\rho/\rho^\circ - (k-1)}{[1 + k(2k/n^\circ - 1)] - (k-1)\,\rho/\rho^\circ} \tag{17}$$

In the case without heat transfer between the phases the exponent n differs very little from k (in our example $n^0/k = 1.0011$), i.e., from the viewpoint of the states at $\pm\infty$ the two-phase medium behaves like a "heavy" gas with adiabatic exponent approximately equal to k.

In this sense the particles, just as in the isothermal case, increase the shock intensity. If heat transfer between the phases is taken into account, the exponent n may differ markedly from k. In this case the process is complicated by thermal relaxation, which intensifies the shock. In this sense the isothermal shock may be considered as the limiting case in which the gas is ideally thermally conductive and the particles are a thermostat.

Returning to the question of the possibility of describing unsteady processes in a two-phase medium with the aid of the steady-state equations, we note the following.

We see from Figs. 1 and 3 that in the case of sufficiently small particles the scale of the process in the gas can be the scale for the medium as a whole, so that steady-state theory is applicable without any limitation.

In the case of larger particles these scales do not coincide. The order of magnitude of the relaxation zone in the simplest case can be evaluated by solving the equation of motion of an individual particle suspended in a gas stream traveling with velocity u (for Stokes interaction):

$$\frac{R^\circ}{9}\frac{\rho_{2i}}{\rho_{1i}}\frac{dU_2}{dX} = \frac{1-U_2}{U_2} \tag{18}$$

Equation (18) is integrated for U = const and with the following boundary condition: $X = 0$, $U_2 = 0$. We obtain

$$X = -\frac{U_2 + \ln(1-U_2)}{(9/R^\circ)(\rho_{1i}/\rho_{2i})} \tag{19}$$

Setting $U_2 \approx 0.667$ (of order $1 - e^{-1}$ according to relaxation theory), it is easy to obtain the value of ΔX, which can be called the relaxation region:

$$\Delta X \approx 0.0481 R^\circ \rho_{2i} / \rho_{1i} \tag{20}$$

Thus the method of stationary processes in a two-phase medium can be used to study only those unsteady processes in which the variation of the macroscopic parameters takes place in regions significantly longer than ΔX. In this case, in calculating R° we must take as the characteristic velocity the magnitude of the velocity jump in the shock wave.

In the present paper we have studied that particular case of a shock in which the state ahead of the shock is in thermal equilibrium and the phase velocities are the same. In the general case the qualitative picture may be considerably more complex.

REFERENCES

1. Kh. A. Rakhmatulin, "Fundamentals of the gasdynamics of interpenetrating motions of compressible media," PMM, vol. 20, no. 2, 1956.

2. Ya. Z. Kleiman, "The propagation of strong discontinuities in a multicomponent medium," PMM, vol. 22, no. 2, 1958.

3. G. Rudinger and A. Chang, "Analysis of nonsteady two-phase flow," Phys. Fluids, vol. 7, no. 11, pt. 1, pp. 1747–1754, 1964.

4. J. D. Hirschfelder, C. F. Curtiss, and R. B. Bird, Molecular Theory of Gases and Liquids [Russian translation], Izd-vo inostr. lit., Moscow, 1961.

5. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena [in Russian], Fizmatgiz, Moscow, 1963.

6. L. I. Sedov, Similarity and Dimensional Methods in Mechanics [in Russian], 5th edition, Nauka, Moscow, 1965.

7. G. N. Patterson, Molecular Flow of Gases [Russian translation], Fizmatgiz, Moscow, 1960.

23 July 1968

Voronezh